Appendix A

Magnetic helicity of a CT

The key idea in magnetic relaxation is that short timescale effects such as microturbulence and reconnection cause the magnetic fields to decay and become reconfigured in space in such a way as to find a state of minimum total energy. This minimum is non-zero because the total magnetic helicity of the plasma is a conserved quantity on the MHD timescales, and this provides a constraint on the system. The magnetic field can only become reconfigured in ways that preserve the value of the magnetic helicity $\mathbf{A} \cdot \mathbf{B}$ integrated over the volume of the plasma. In order to measure this constant

$$H = \int_V \mathbf{A} \cdot \mathbf{B} \ \mathbf{dV} \tag{A.1}$$

we need to determine the vector potential $\mathbf{A}$. The magnetic field is curl of the vector potential, and so A is determined only up to the gradient of a scalar function.

For the case of a force-free field it is sufficiently general to formulate A in terms of the magnetic field and two scalar functions $\gamma = \gamma(\mathbf{x})$ and $\psi = \psi(\mathbf{x})$ according to

$$\mathbf{A} = \gamma \mathbf{B} + \nabla \psi \tag{A.2}$$

The helicity is gauge invariant for a bounded plasma in a conducting vessel, and the helicity integral

simplifies to

$$\int_V \mathbf{A} \cdot \mathbf{B}\ \mathbf{dV} = \int_V \gamma B^2\ dV$$

The $\nabla \psi$ term was delt with by applying a vector identity in combination with $\nabla \cdot \mathbf{B} = 0$, followed by the use of the divergence theorem and the fact that$\mathbf{B} \cdot \mathbf{n} = \mathbf{0}$ at the walls of the conducting vessel. We see that the grad ψ term vanishes.

$$\int_V \nabla \psi \cdot \mathbf{B}\ \mathbf{dV} = \int_V \nabla \cdot (\psi \mathbf{B})\ \mathbf{dV} = \int_{\partial V} \psi \mathbf{B} \cdot \mathbf{dA} = 0$$

For a force free magnetic field configuration satisfying $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ the multiplicative factor $\gamma(\mathbf{x})$ that determines $\mathbf{A}$ can be found by taking the curl of $\mathbf{A}$

$$\mathbf{B} = \nabla \times \mathbf{A} = \gamma \nabla \times \mathbf{B} + (\nabla \gamma) \times \mathbf{B}$$

so

$$\lambda \mathbf{B} = \frac{1}{\gamma} \mathbf{B} - \frac{\nabla \gamma}{\gamma} \times \mathbf{B}$$

Then taking the dot product with $\mathbf{B}$ eliminates the cross product term and we see that

$$\lambda B^2 = \frac{1}{\gamma(\mathbf{x})} B^2$$

so $\gamma(\mathbf{x}) = \mathbf{1}/\lambda$ which is a constant. So the vector potential is uniquely determined (up to a scalar gauge) by the magnetic field and the force free eigenvalue λ. The vector potential for a force free magnetic field is simply

$$\mathbf{A} = \frac{\mathbf{1}}{\lambda} \mathbf{B} \tag{A.3}$$

The magnetic helicity is then directly proportional the total magnetic energy

$$H = \frac{1}{\lambda} \int_V B^2\ dV \tag{A.4}$$

This result implies that once a plasma has reached a force-free state, the total magnetic energy can not be reduced any further because it is constrained by the conservation of H.From then on, the magnetic field and the helicity can decay only at a much slower rate due to the bulk resistivity of the plasma. [ref A.M. Dixon Astron. Astrophys. 225, 156-166 (1989)]

Appendix B

Magnetic computations for probe calibration

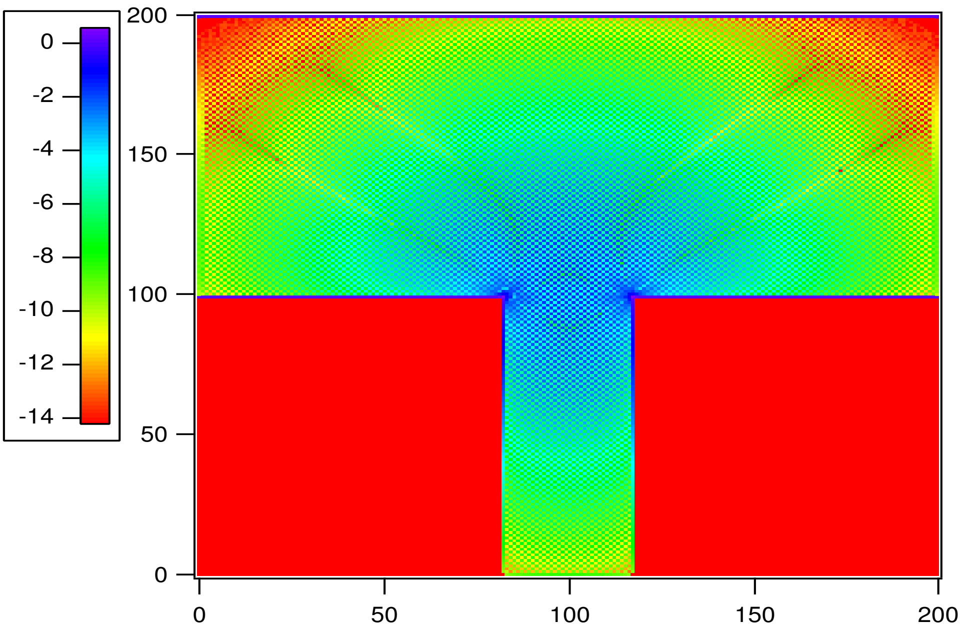

Figure B.1: The log (base 10) of the value of the Laplacian of the magnetostatic potential near a conducting port well. Since the goal is $\nabla^2 \phi = 0$, this graph indicates the order of magnitude of the error in the calculation.